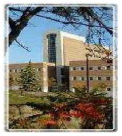


Fleet Maintenance Simulation With Insufficient Data

Zissimos P. Mourelatos

Mechanical Engineering Department

Oakland University

mourelat@oakland.edu

Ground Robotics Reliability Center (GRRC) Seminar
17 March 2010

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **17 MAR 2010**	2. REPORT TYPE **Briefing Charts**	3. DATES COVERED **04-03-2009 to 26-02-2010**

4. TITLE AND SUBTITLE **Fleet Maintenance Simulation With Insufficient Data**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) **Zissimos Mourelatos**	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Mechanical Engineering Department,Oakland University,2200 N. Squirrel Road,Rochester,Mi,48309**	8. PERFORMING ORGANIZATION REPORT NUMBER **; #20590**
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) **U.S. Army TARDEC, 6501 East Eleven Mile Rd, Warren, Mi, 48397-5000**	10. SPONSOR/MONITOR’S ACRONYM(S) **TARDEC**
	11. SPONSOR/MONITOR’S REPORT NUMBER(S) **#20590**

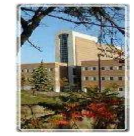
12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
For Ground Robotics Reliability Center (GRRC) Seminar 17 March 2010

14. ABSTRACT
briefing charts

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Public Release**	**54**	

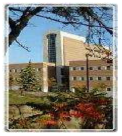


Goal

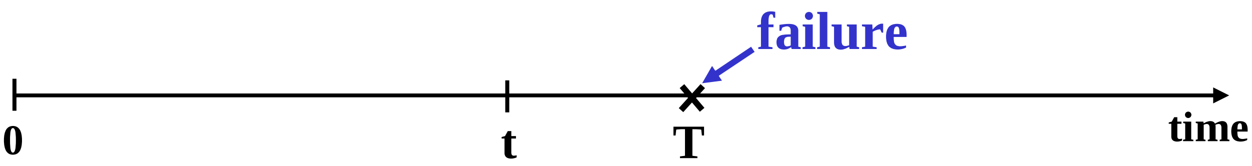
➢ **Apply time-dependent reliability/durability concepts to address prognostic CBM using**

- **Available data (limited, censored)**
- **“Expert” opinion**
- **Computer simulations (physics-of-failure data)**

What is Reliability?



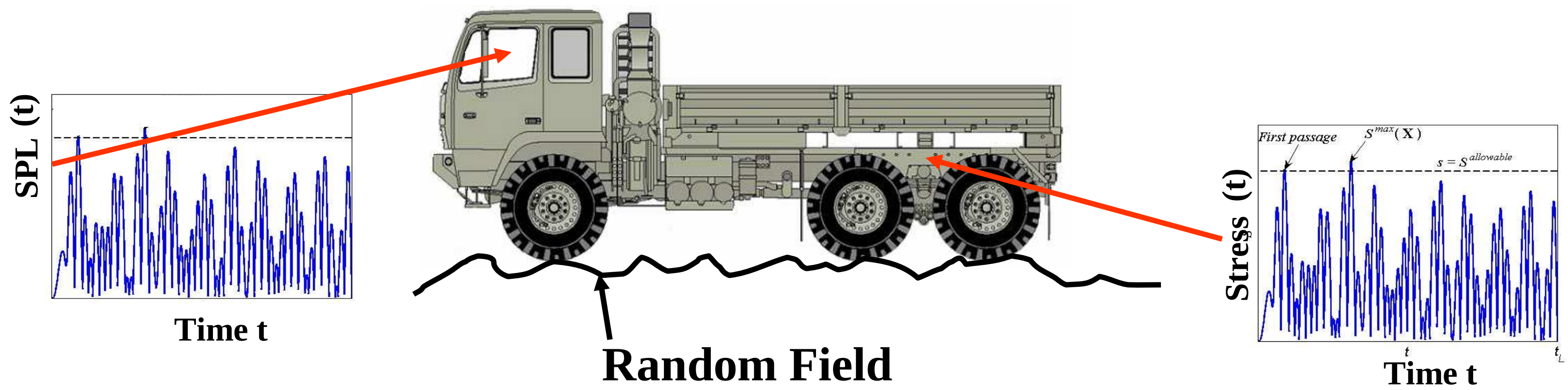
Reliability at time t is the probability that the system has not failed before time t.



$$\boxed{R(t) = P(T > t) = 1 - P(T \le t)}$$

Time-Dependent Reliability

Background Information



Response(t) = f [E(t), Degradation/Wear(t), Load(t)]

Random Process approach to reliability-based design is needed → time-dependent reliability

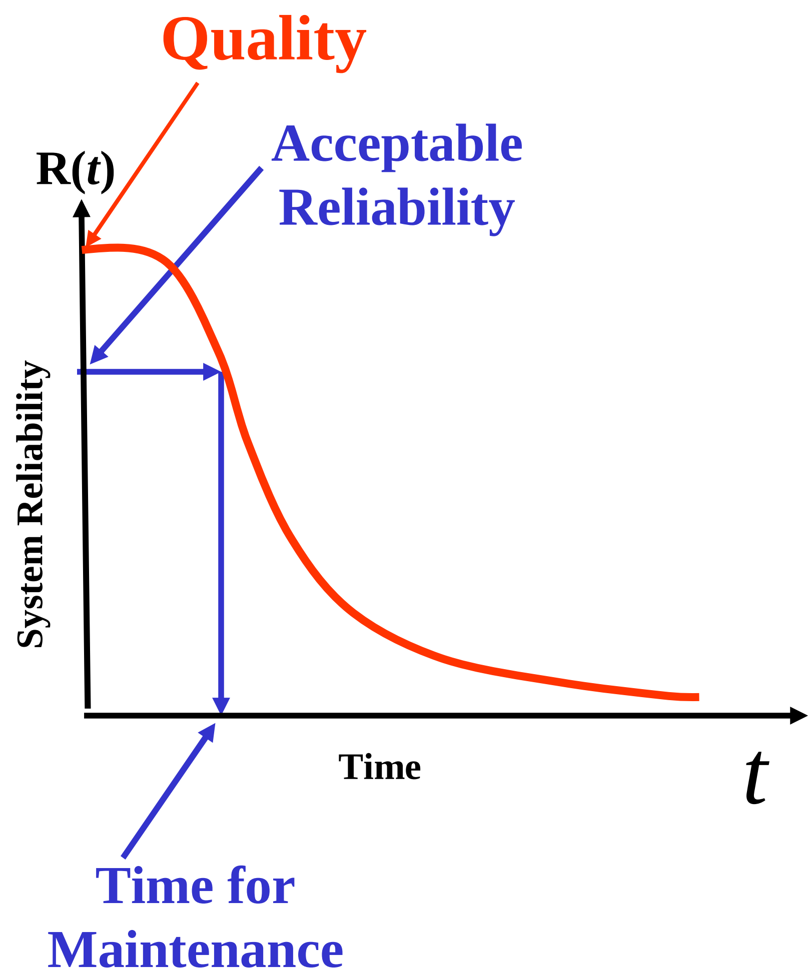
$$\boxed{\textbf{Limit States: } \mathbf{g}\left(\mathbf{X}(t), \mathbf{Y}, t\right)}$$

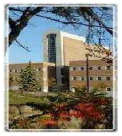
Random Process ($\mathbf{X}(t)$)

Random Variable ($\mathbf{Y}$)

What can we Get from Time-Dependent Reliability?

- ➢ **Define lifecycle cost and design for it.**
- ➢ **Use R(t) in CBM to determine "time to maintenance."**
- ➢ **Design for:**
  - **Lifecycle cost**
  - **Quality**
  - **Warranty**
  - **Maintenance schedule**





Definitions / Observations

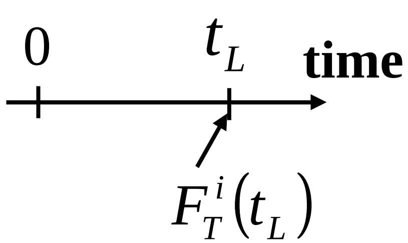
Reliability: **Ability of a system to carry out a function in a time period $[0, t_L]$**

$$p_f^c = P(t \le t_L) = F_T^c(t_L)$$ **Prob. of Time to Failure**

$$\boxed{F_T^c(t_L) = P\left(\exists t \in [0, t_L], \textit{such} \quad \textit{that} \quad g(\mathbf{X}(t), t) \le 0\right)}$$ **Cumulative Prob. of Failure**

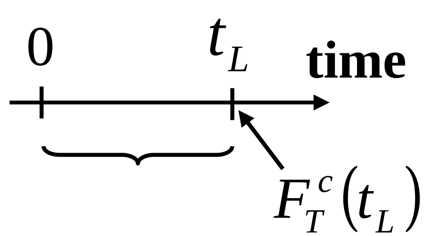
$$\boxed{F_T^i(t_L) = P\left(g(\mathbf{X}(t_L), t_L) \le 0\right)}$$ **Instantaneous Prob. of Failure**

Time-Invariant Reliability



$\neq$

Time-Variant Reliability



Design for Lifecycle Cost

Lifecycle Cost = Production Cost

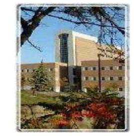
+ Inspection Cost

+ Expected Variable Cost

Quality

Time-Dependent System Reliability

Accurate and efficient predictive tools are needed to estimate **Time-dependent System Reliability**



Design for Lifecycle Cost

$$C_L\left(\mathbf{d},\mathbf{X},t_f,r\right)=C_P\left(\mathbf{d},\mathbf{X}\right)+C_I\left(\mathbf{d},\mathbf{X},t_0\right)+C_V^E\left(\mathbf{d},\mathbf{X},t_f,r\right)$$

Lifecycle Cost **Production Cost** **Inspection Cost** **Expected Variable Cost**

Final time **Interest rate**

$$C_V^E\left(\mathbf{d},\mathbf{X},t_f,r\right)=\int_0^{t_f} c_F(t)\,e^{-rt}\,f_T^c(t)\,dt$$

Cost of failure at time t **PDF of time to failure time**

$$F_T^c\left(t_L\right)=P\left(\exists t\in\left[0,t_L\right], such \quad that \quad g\left(\mathbf{X}(t),t\right)\leq 0\right)$$

How Can we Use it in Design?

➤ **Specify a Desired System Reliability in Time**

$$\min_{\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\sigma}_{\mathbf{X}}} C_L\left(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\sigma}_{\mathbf{X}},t_f,r\right)$$

$$\textbf{s. t. } F_Q\left(\mathbf{d},\mathbf{X},t_0\right)\leq p_f^t\left(t_0\right)$$

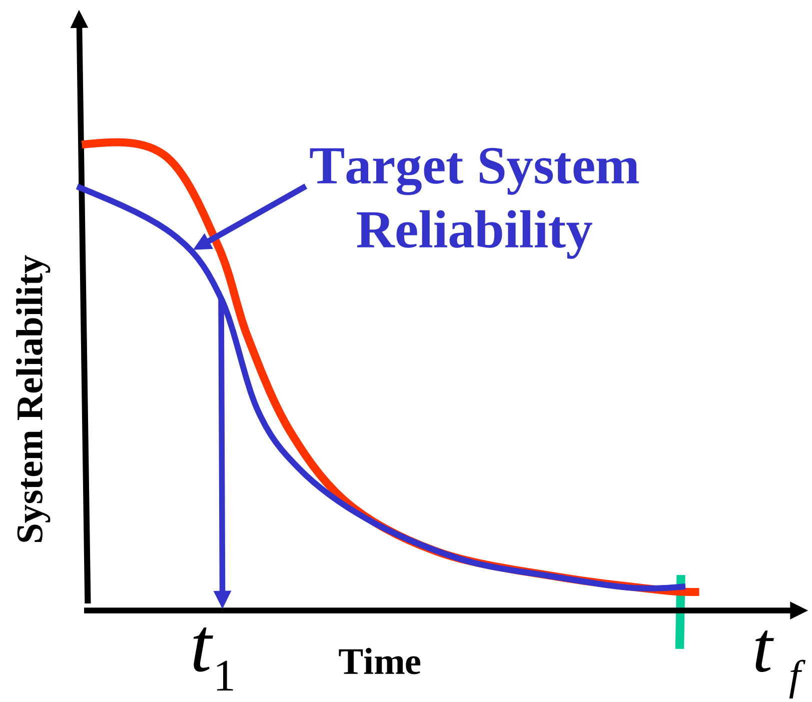
$$F_R^c\left(\mathbf{d},\mathbf{X},t_1\right)\leq p_f^t\left(t_1\right)$$

$$F_R^c\left(\mathbf{d},\mathbf{X},t_f\right)\leq p_f^t\left(t_f\right)$$

$$\mathbf{d}_L\leq\mathbf{d}\leq\mathbf{d}_U$$

$$\boldsymbol{\mu}_{\mathbf{X}_L}\leq\boldsymbol{\mu}_{\mathbf{X}}\leq\boldsymbol{\mu}_{\mathbf{X}_U}$$

$$\boldsymbol{\sigma}_{\mathbf{X}_L}\leq\boldsymbol{\sigma}_{\mathbf{X}}\leq\boldsymbol{\sigma}_{\mathbf{X}_U}$$



How Can we Use it in Design?

➢ **Determine Optimal Time to Maintenance in CBM**

$$\max_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X},\boldsymbol{\sigma}_\mathbf{X}} t_M$$

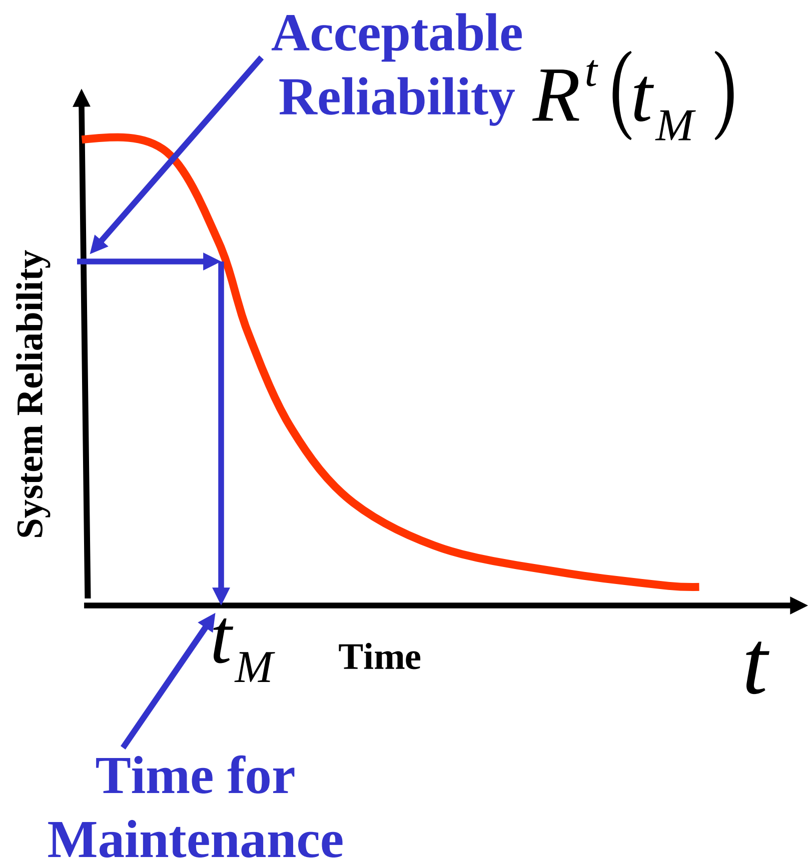
$$\textbf{s. t.}\quad C_L\left(\mathbf{d},\boldsymbol{\mu}_\mathbf{X},\boldsymbol{\sigma}_\mathbf{X},t_M,r\right)\le C_L^t$$

$$F_R^c\left(\mathbf{d},\mathbf{X},t_M\right)\le 1-R^t\left(t_M\right)$$

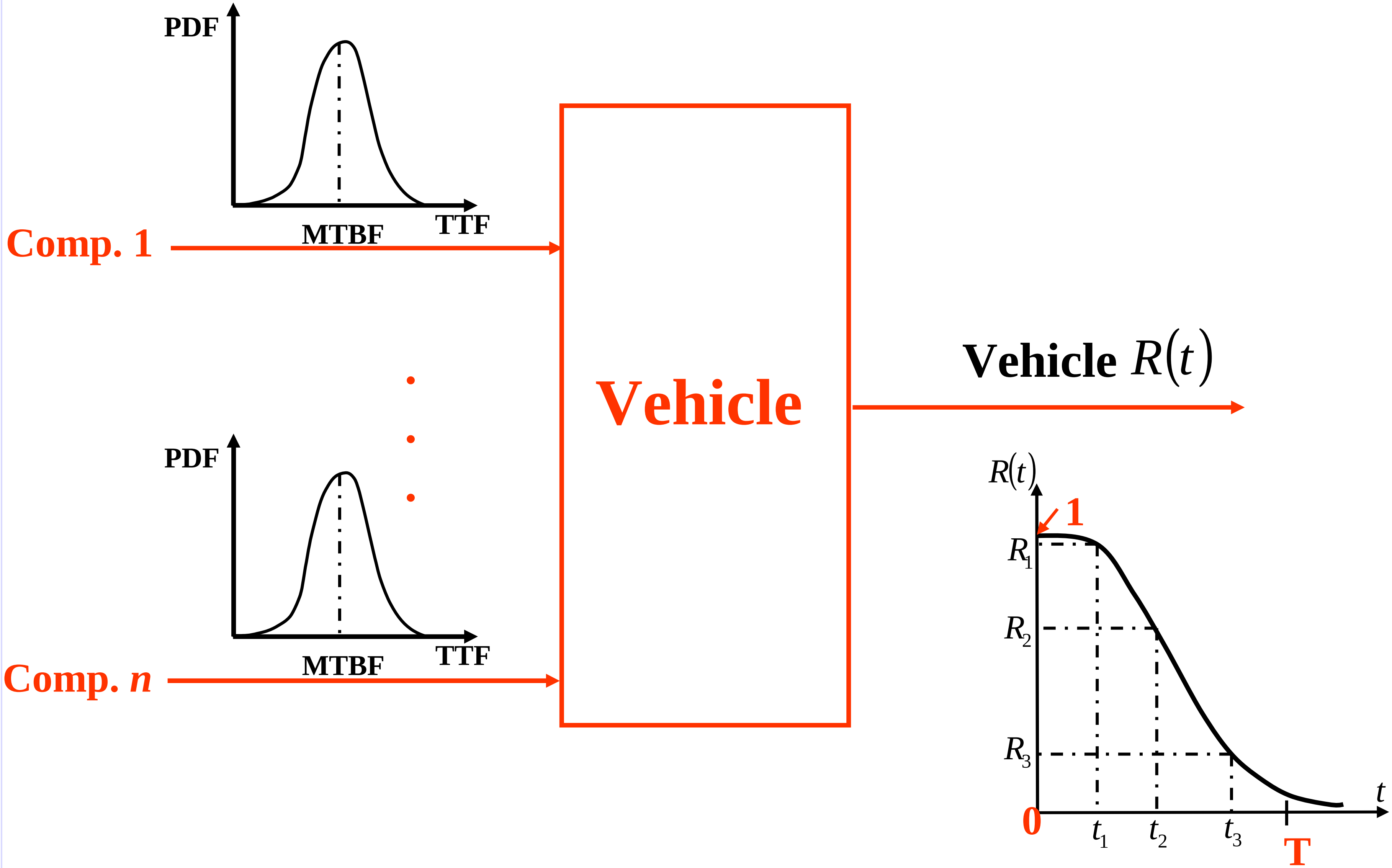
$$\mathbf{d}_L\le\mathbf{d}\le\mathbf{d}_U$$

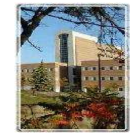
$$\boldsymbol{\mu}_{\mathbf{X}_L}\le\boldsymbol{\mu}_\mathbf{X}\le\boldsymbol{\mu}_{\mathbf{X}_U}$$

$$\boldsymbol{\sigma}_{\mathbf{X}_L}\le\boldsymbol{\sigma}_\mathbf{X}\le\boldsymbol{\sigma}_{\mathbf{X}_U}$$



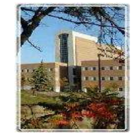
System (Vehicle) Reliability



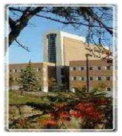


We Need

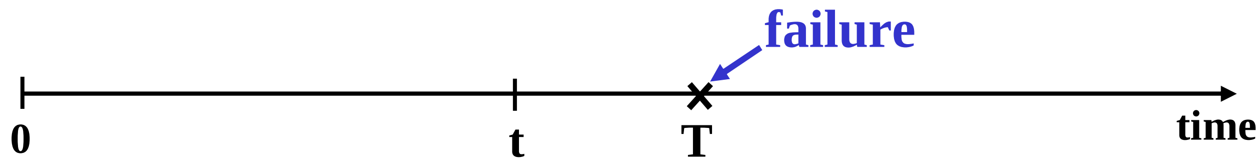
- **A Tool to Estimate the PDF of Time Between Failures (TBF) using limited, censored data**
  - **"Frequentist" approach (Method 1)**
  - **Bayesian updating approach (Method 2)**
    - ✓ **"Enhances" data with expert opinion**
- **A Tool to Estimate System (Vehicle) Reliability**
  - **Monte Carlo Simulation**



Reliability Basics for Non-Repairable Systems



Reliability of Non-Repairable Systems

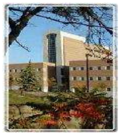


$$R(t) = P(T > t) = 1 - P(T \le t) \Rightarrow \boxed{R(t) = 1 - F(t)} \qquad \textbf{(1)}$$

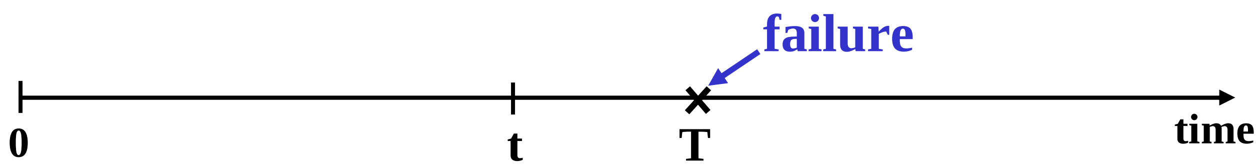
$$\lambda(t) = \frac{P(t < T \le t + dt / T > t)}{dt} = \frac{P(t < T \le t + dt)}{dt * P(T > t)} =$$

Failure Rate

$$= \frac{F(t+dt) - F(t)}{dt * R(t)} \Rightarrow \boxed{\lambda(t) = \frac{f(t)}{R(t)}} \qquad \textbf{(2)}$$



Reliability of Non-Repairable Systems



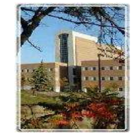
due to (2)

$$R(t)=1-F(t)\Rightarrow\frac{dR}{dt}=-f(t)\Rightarrow\frac{dR}{dt}=-\lambda(t)R(t)\Rightarrow$$

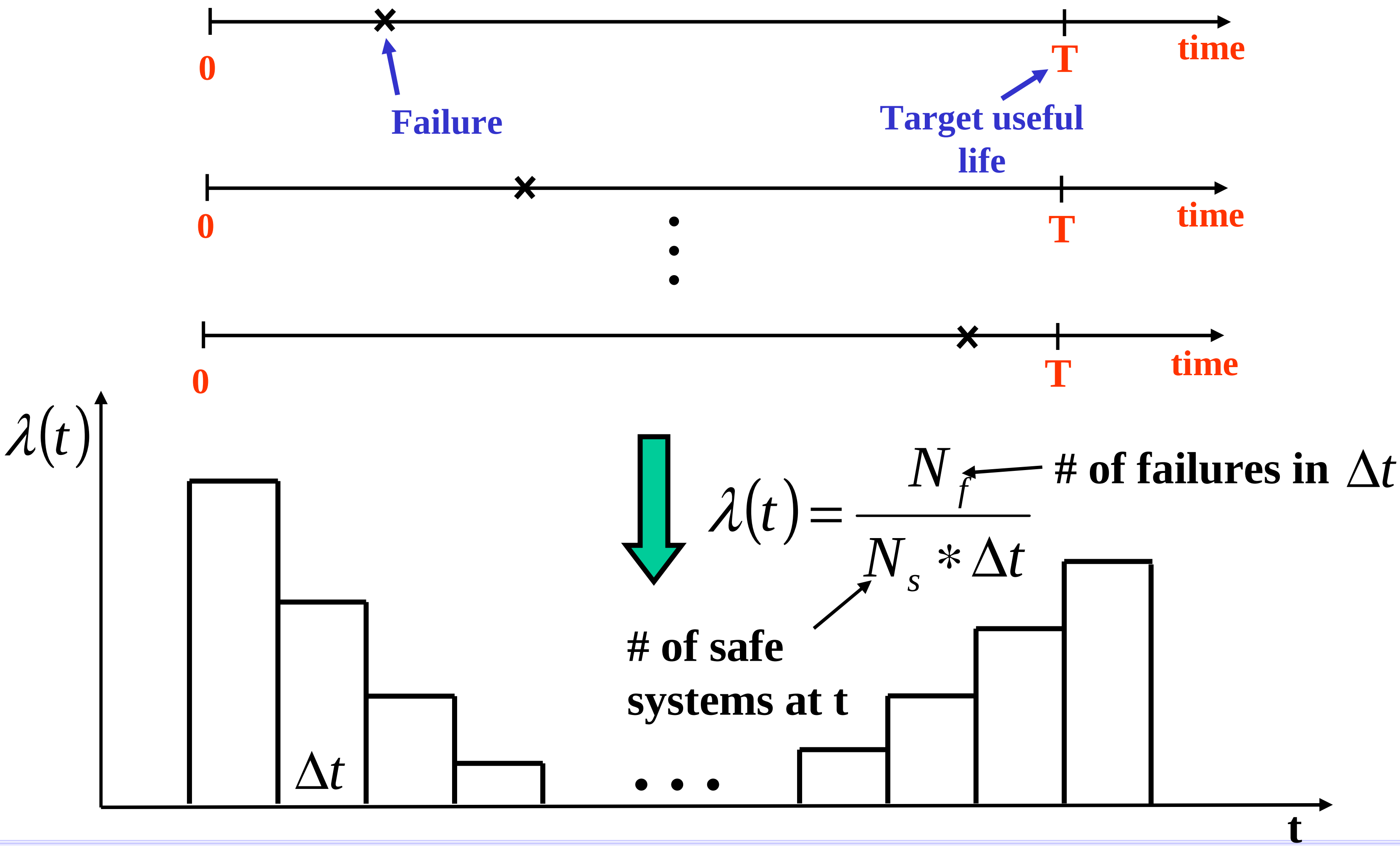
$$\Rightarrow\frac{dR}{R}=-\lambda dt\Rightarrow d(\ln R)=-\lambda dt\Rightarrow\ln\left(\frac{R(t)}{\cancelto{1}{R(0)}}\right)=-\int_0^t\lambda dt\Rightarrow$$

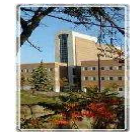
$$\Rightarrow\boxed{R(t)=\exp[-\int_0^t\lambda dt]}$$

All we need is the failure rate



Reliability of Non-Repairable Systems



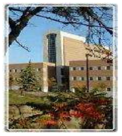


Reliability Calculation

All we need for calculating the reliability of a system (non-repairable or repairable) is the system PDF of time to failure (TTF)

We use :

- **Data to estimate the PDF of TTF for each component**
- **Monte Carlo simulation to estimate the PDF of TTF for the system**



Estimation of the PDF (or CDF) of the TTF (TBF) using Limited, Censored Data

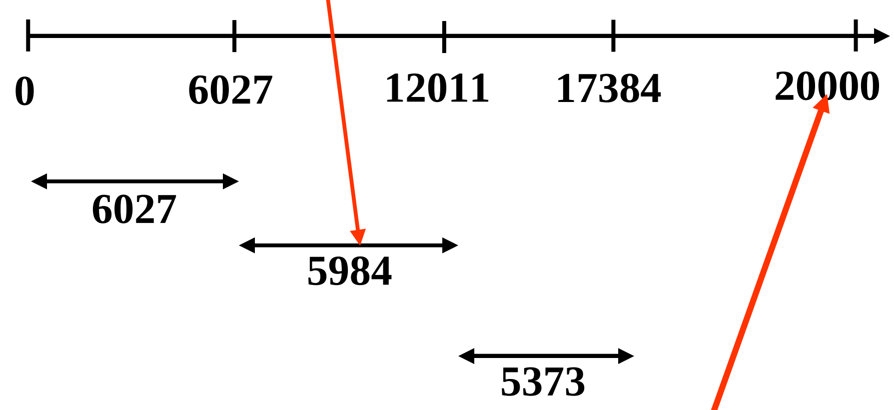
- **Two approaches will be presented:**
  - Censored MLE approach (Method 1)
  - Bayesian updating approach (Method 2)
    - ✓ "Enhances" data with expert opinion

Limited Data

Group L1

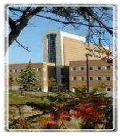
Original data		Updated data	
Vehicle#	**mileage**	**Vehicle#**	**mileage**
10	741	1	10247
4	5273	2	9044
7	6027	2	8977
5	7398	3	13984
6	7495	3	4064
2	9044	4	5273
1	10247	4	9747
8	12008	5	7398
7	12011	5	7611
9	12014	6	7495
10	12074	6	7516
3	13984	7	6027
5	15009	7	5984
6	15011	7	5373
4	15020	8	12008
7	17384	9	12014
2	18021	10	741
3	18048	10	11333

Time Between Failures (TBF)

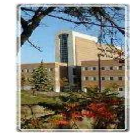


Censoring Mileage

Censored MLE Approach (Method 1)



- Using available limited data (TBFs and censoring mileage), "estimate" PDF of TBF using a censored MLE approach.
- Tail sample the PDF of previous step to "enhance" the original limited data.
- Using "enhanced" data from previous step, "better estimate" the PDF of TBF using an uncensored MLE approach.
- Using the PDF of previous step, a Bootstrap approach estimates statistics of TBF (e.g. distribution of MTBF, distribution of TBF standard deviation, etc.)



Bayesian Updating Approach (Method 2)

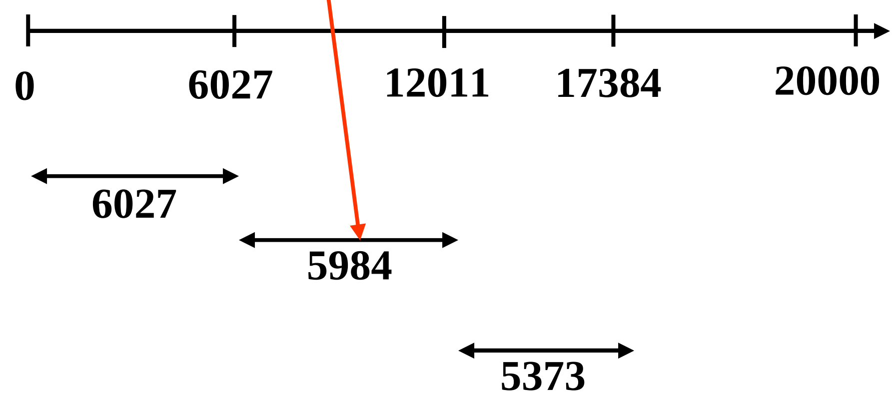
- Use a Bayesian approach to estimate statistics of TBF (e.g. distribution of MTBF, distribution of TBF standard deviation, etc.). The Bayesian approach:
  - Refines estimate by progressively collecting data on a "as needed" basis.
  - Allows fusion of available data with "expert" opinion.

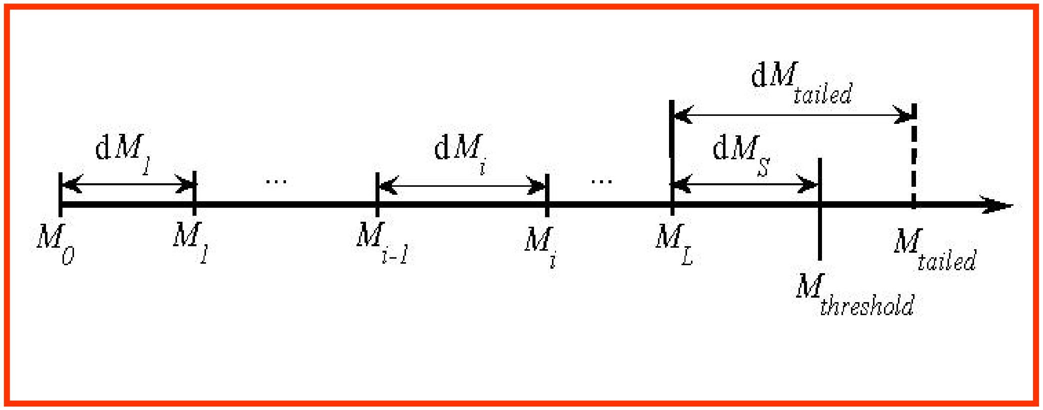
Notation

Group L1

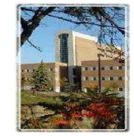
Original data Vehicle#	Original data mileage	Updated data Vehicle#	Updated data mileage
10	741	1	10247
4	5273	2	9044
7	6027	2	8977
5	7398	3	13984
6	7495	3	4064
2	9044	4	5273
1	10247	4	9747
8	12008	5	7398
7	12011	5	7611
9	12014	6	7495
10	12074	6	7516
3	13984	7	6027
5	15009	7	5984
6	15011	7	5373
4	15020	8	12008
7	17384	9	12014
2	18021	10	741
3	18048	10	11333

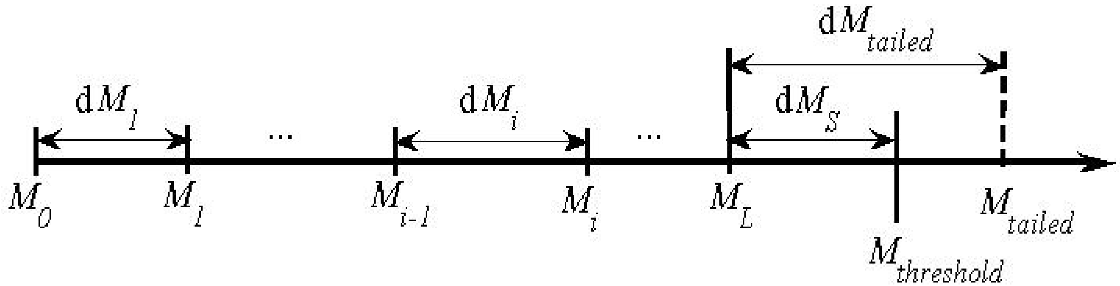
Time Between Failures (TBF)





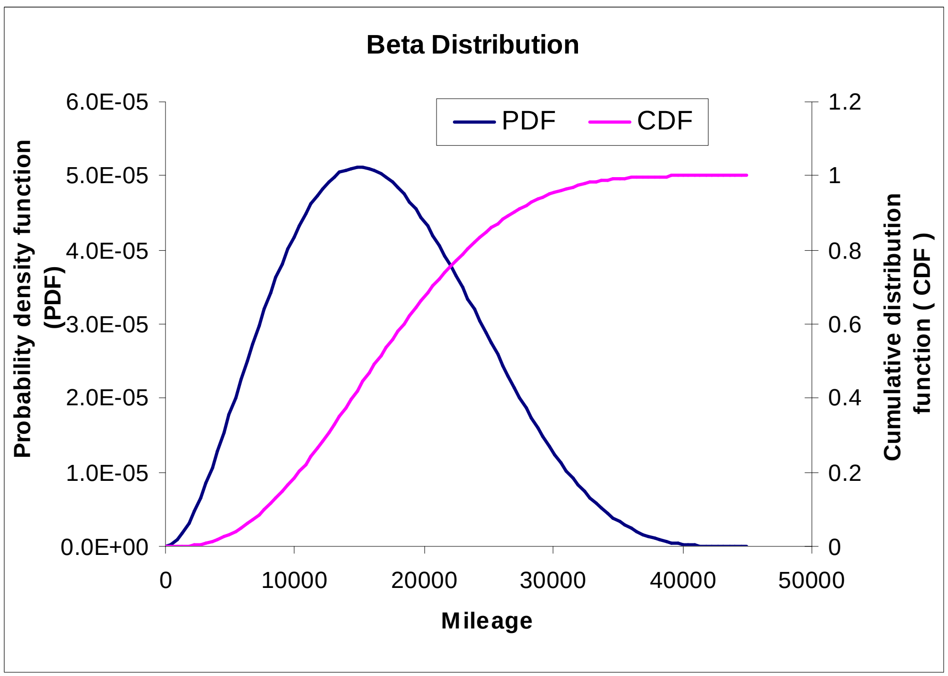
Observation / Assumption





$$\mathrm{d}M_i = X_i \sim \beta(A, B, p, q), \quad (A \le X_i \le B, \text{ and } p > 0, q > 0)$$

$$f(x, A, B, p, q) = \beta(p, q)^{-1} (x - A)^{p-1} (B - x)^{q-1} / (B - A)^{p+q-1}, \quad (A \le x \le B, \textit{ and } p > 0, q > 0)$$



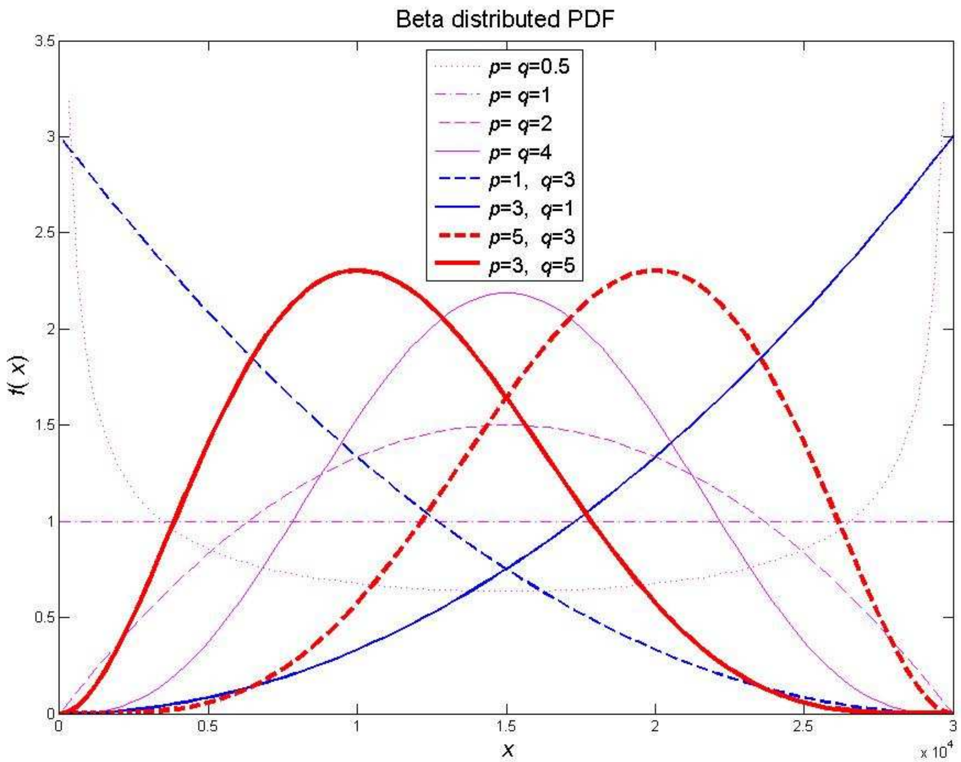
A = 0

B = 45,000 miles

p = 3, q = 5

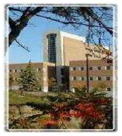
Observation / Assumption

- **Beta distribution family is used to model TBF.**



$$A=0,\ B=30000$$

$$f(x,A,B,p,q)=\beta(p,q)^{-1}(x-A)^{p-1}(B-x)^{q-1}/(B-A)^{p+q-1},\quad (A \le x \le B, \text{ and } p>0,\ q>0)$$



MLE Approach

Determines parameters (*A, B, p, q*) of "most likely" Beta distribution using available data. It provides Likelihood function in Bayesian estimation.

Censored MLE

# of recorded failures **# of survivals**

$$\underset{A,B,p,q}{Max} \prod_{i=1}^{N_F} f\left(x_i, A, B, p, q\right) \prod_{j=1}^{N_s} \left[1 - F\left(x_j, A, B, p, q\right)\right]$$

Beta PDF **Beta CDF**

Uncensored MLE

$$\underset{A,B,p,q}{Max} \prod_{i=1}^{N} f\left(x_i, A, B, p, q\right)$$

Bayesian Updating

- **Progressively updates estimated Beta parameters (*A, B, p, q*) using prior knowledge and available new data.**
- **It allows to "fuse" available data with expert opinion.**

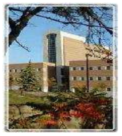
$$\boxed{Posterior(\boldsymbol{\theta}) \propto L(\boldsymbol{\theta} / DATA) * \Pr ior(\boldsymbol{\theta})}$$ **with** $\boldsymbol{\theta} = \{A \quad B \quad p \quad q\}$

where:

$$DATA = \{\underbrace{DATA_F}_{\text{failures}} \quad \underbrace{DATA_S}_{\text{survivals}}\}$$

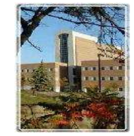
and

$$L(\boldsymbol{\theta} / DATA) = L(\boldsymbol{\theta} / DATA_F)L(\boldsymbol{\theta} / DATA_S) = \prod_{i=1}^{N_F} f(x_i, \boldsymbol{\theta}) \prod_{j=1}^{N_S} [1 - F(x_j, \boldsymbol{\theta})]$$



Censored MLE Approach (Method 1)

1. Enter recorded failure data
2. Data sorting
3. Histogram of recorded failure data
4. Maximum Likelihood Estimation (MLE) with censored data
5. Tail sampling to get inferred failure mileage
6. Histogram of both recorded and tailed failure data
7. MLE with uncensored data (considering tailed data)
8. Failure probability bounds are calculated by Bootstrap method



Censored MLE Approach (Method 1)

1. Enter recorded failure data

- **Artificial data used: 15 vehicles, 4 tires each side,**
- **$M_{\text{threshold}}$ =30,000 miles**
- **Beta distribution: A=0, B=45,000, p = 3, and q = 5**

$$\mathrm{d}M_i = X_i \sim \beta(A, B, p, q), \quad (A \le X_i \le B, \text{and } p > 0, q > 0)$$

	A	B	C	D	E	F	G	H	I	J	K	L
1	PROCEDURE	Method		Counts		Counts		Buttons	Options		Survivals	
2		1		161		0		Data Sorting	0			
3								Tail Sampling	0			
4	$M_{\text{threshold}}$	30000	Recorded Failure Data					Sorted Failure Data			Survival Data	
5	Vehicle No.	Tire Location	Odometer Mileage	Failure Mode		Vehicle	Tire	Odometer Mileage	Failure Mode	Failure Mileage	Survival Mileage	Tailed Failure Mileage
6	7	L4	21764.88086	WO								
7	4	R1	25169.7207	WO								
8	12	R2	19132.91602	WO								
9	6	L1	18305.94727	WO								
10	5	R2	28231.19336	WO								
11	5	L3	10868.71875	WO								
12	15	L3	19211.23633	WO								
13	8	R3	14433.77148	WO								
14	13	L4	10622.08398	WO								
15	10	L2	11497.66406	WO								
16	1	R4	13365.61914	RH								
17	12	L4	19039.30664	WO								

Demo Data / Failure Data / Data Analysis / Beta Dist / Resampling Tab / Bayesian Updating / GA Solver / output generations / warnings

Censored MLE Approach (Method 1)

2. Data sorting

- ➢ Sort recorded failure data (white cells)
- ➢ Retrieve “failure mileage” data (164) and “survival mileage” data (120)

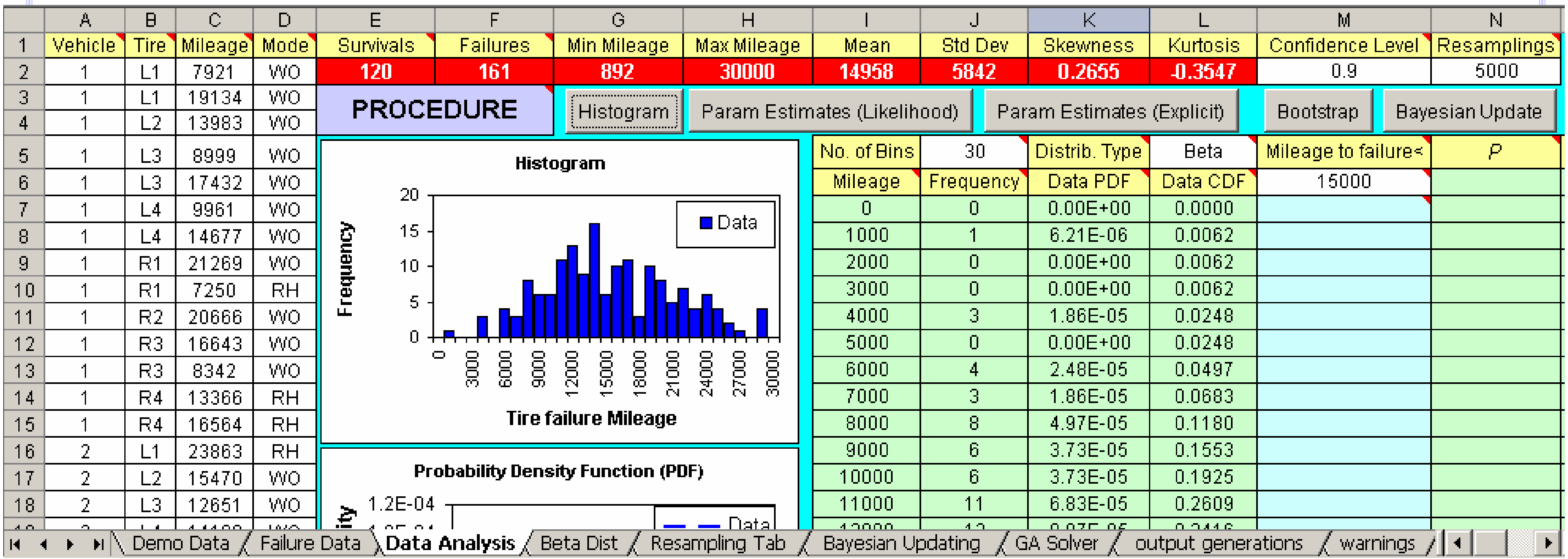
	A	B	C	D	E	F	G	H	I	J	K	L
1	PROCEDURE	Method		Counts		Counts		Buttons	Options		Survivals	
2		1		161		164		Data Sorting	0		120	
3								Tail Sampling	0			
4	$M_{threshold}$	30000	Recorded Failure Data			Sorted Failure Data					Survival Data	
5	Vehicle No.	Tire Location	Odometer Mileage	Failure Mode		Vehicle	Tire	Odometer Mileage	Failure Mode	Failure Mileage	Survival Mileage	Tailed Failure Mileage
6	7	L4	21764.88086	WO		1	L1	7921	WO	7921		
7	4	R1	25169.7207	WO		1	L1	27055	WO	19134	2945	
8	12	R2	19132.91602	WO		1	L2	13983	WO	13983	16017	
9	6	L1	18305.94727	WO		1	L3	8999	WO	8999		
10	5	R2	28231.19336	WO		1	L3	26431	WO	17432	3569	
11	5	L3	10868.71875	WO		1	L4	9961	WO	9961		
12	15	L3	19211.23633	WO		1	L4	24638	WO	14677	5362	
13	8	R3	14433.77148	WO		1	R1	21269	WO	21269		
14	13	L4	10622.08398	WO		1	R1	28519	RH	7250	1481	
15	10	L2	11497.66406	WO		1	R2	20666	WO	20666	9334	
16	1	R4	13365.61914	RH		1	R3	16643	WO	16643		
17	12	L4	19039.30664	WO		1	R3	24985	WO	8342	5015	

Demo Data / **Failure Data** / Data Analysis / Beta Dist / Resampling Tab / Bayesian Updating / GA Solver / output generations / warnings

Censored MLE Approach (Method 1)

3. Histogram of recorded failure data

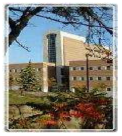
- Considers failure mileage data
- DOES NOT consider survival mileage data
- Histogram shape may change with different number of bins and ranges
- Histogram, PDF, and CDF of the failure data

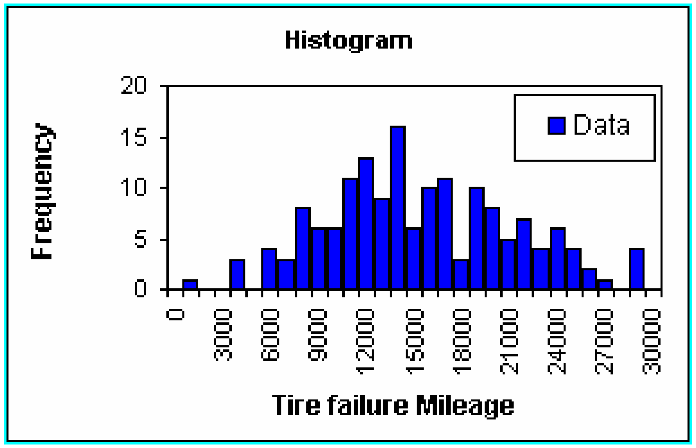


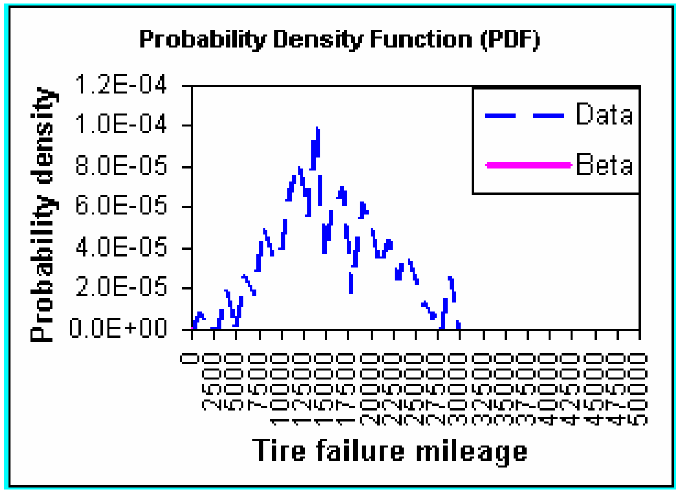
	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Vehicle	Tire	Mileage	Mode	Survivals	Failures	Min Mileage	Max Mileage	Mean	Std Dev	Skewness	Kurtosis	Confidence Level	Resamplings
2	1	L1	7921	WO	120	161	892	30000	14958	5842	0.2655	-0.3547	0.9	5000
3	1	L1	19134	WO	PROCEDURE		Histogram	Param Estimates (Likelihood)		Param Estimates (Explicit)			Bootstrap	Bayesian Update
4	1	L2	13983	WO										
5	1	L3	8999	WO					No. of Bins	30	Distrib. Type	Beta	Mileage to failure<	P
6	1	L3	17432	WO					Mileage	Frequency	Data PDF	Data CDF	15000	
7	1	L4	9961	WO					0	0	0.00E+00	0.0000		
8	1	L4	14677	WO					1000	1	6.21E-06	0.0062		
9	1	R1	21269	WO					2000	0	0.00E+00	0.0062		
10	1	R1	7250	RH					3000	0	0.00E+00	0.0062		
11	1	R2	20666	WO					4000	3	1.86E-05	0.0248		
12	1	R3	16643	WO					5000	0	0.00E+00	0.0248		
13	1	R3	8342	WO					6000	4	2.48E-05	0.0497		
14	1	R4	13366	RH					7000	3	1.86E-05	0.0683		
15	1	R4	16564	RH					8000	8	4.97E-05	0.1180		
16	2	L1	23863	RH					9000	6	3.73E-05	0.1553		
17	2	L2	15470	WO					10000	6	3.73E-05	0.1925		
18	2	L3	12651	WO					11000	11	6.83E-05	0.2609		

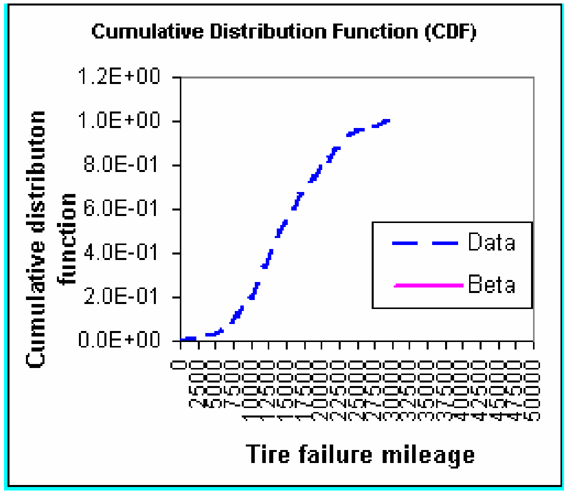
Demo Data / Failure Data / Data Analysis / Beta Dist / Resampling Tab / Bayesian Updating / GA Solver / output generations / warnings

Censored MLE Approach (Method 1)





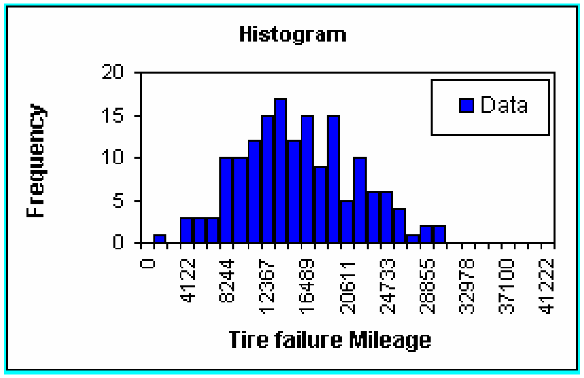


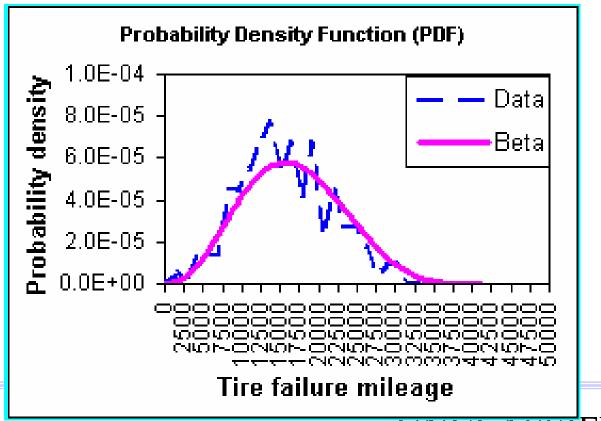


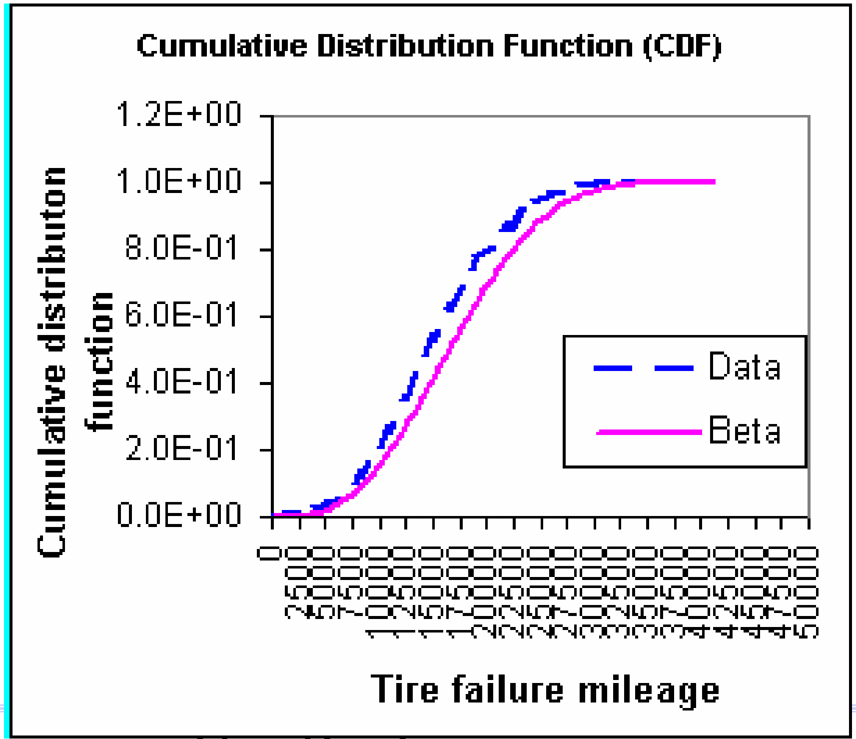
Censored MLE Approach (Method 1)

4. Maximum Likelihood Estimation (MLE) with <u>censored</u> data

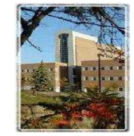
- Considers failure mileage data
- CONSIDERS survival mileage data as “censored” data
- The beta distributed CDF by MLE with censored data, shows that the CDF without survival mileage data is left-biased







Censored MLE Approach (Method 1)



5. Tail sampling to get inferred failure mileage

- Tailed failure mileage data represents inferred failure mileage data of the “survived” tires

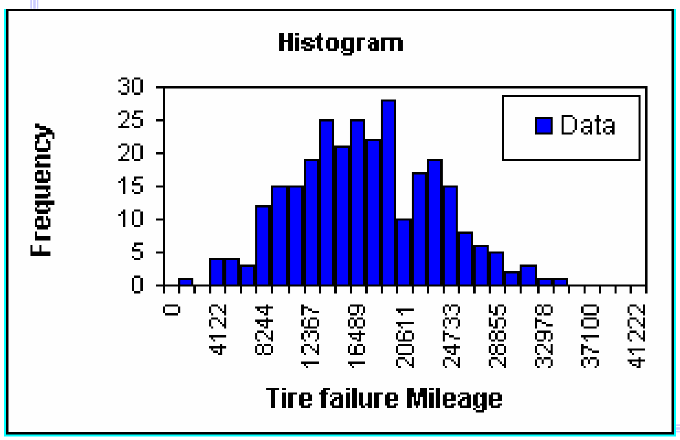
	A	B	C	D	E	F	G	H	I	J	K	L
1	PROCEDURE	Method		Counts		Counts		Buttons	Options		Survivals	
2		1		161		164		Data Sorting	0		120	
3								Tail Sampling	0			
4	$M_{threshold}$	30000	Recorded Failure Data			Sorted Failure Data					Survival Data	
5	Vehicle No.	Tire Location	Odometer Mileage	Failure Mode		Vehicle	Tire	Odometer Mileage	Failure Mode	Failure Mileage	Survival Mileage	Tailed Failure Mileage
6	7	L4	21764.88086	WO		1	L1	7921	WO	7921		
7	4	R1	25169.7207	WO		1	L1	27055	WO	19134	2945	15585
8	12	R2	19132.91602	WO		1	L2	13983	WO	13983	16017	24522
9	6	L1	18305.94727	WO		1	L3	8999	WO	8999		
10	5	R2	28231.19336	WO		1	L3	26431	WO	17432	3569	9610
11	5	L3	10868.71875	WO		1	L4	9961	WO	9961		
12	15	L3	19211.23633	WO		1	L4	24638	WO	14677	5362	12254
13	8	R3	14433.77148	WO		1	R1	21269	WO	21269		
14	13	L4	10622.08398	WO		1	R1	28519	RH	7250	1481	19679
15	10	L2	11497.66406	WO		1	R2	20666	WO	20666	9334	22790
16	1	R4	13365.61914	RH		1	R3	16643	WO	16643		
17	12	L4	19039.30664	WO		1	R3	24985	WO	8342	5015	10492

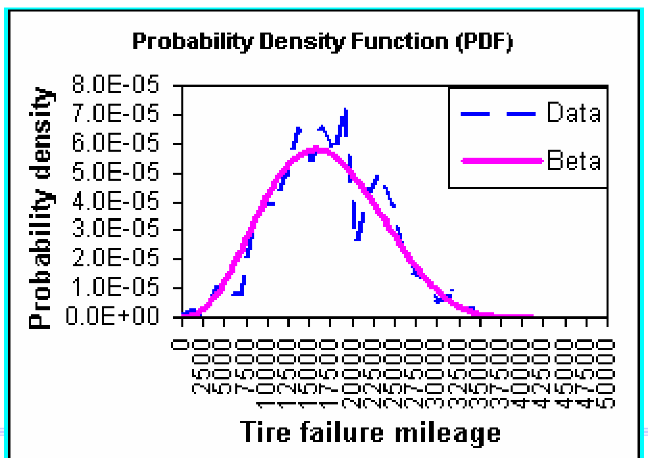
Demo Data / Failure Data / Data Analysis / Beta Dist / Resampling Tab / Bayesian Updating / GA Solver / output generations / warnings

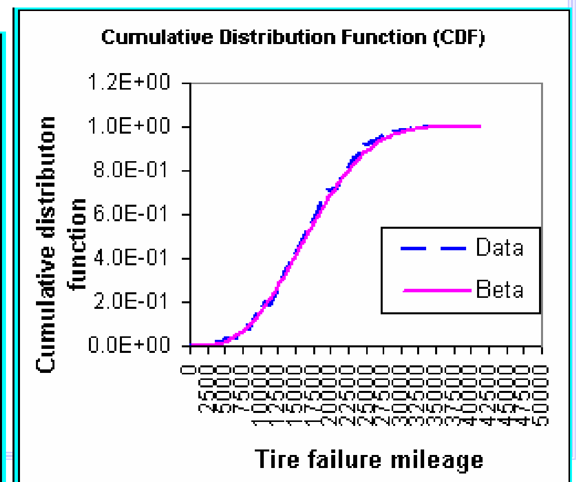
Censored MLE Approach (Method 1)

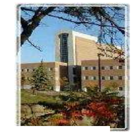
6. Histogram of both recorded and tailed failure data

- **Includes failure mileage data**
- **Includes also tailed failure mileage data**
- **The "tailed" samples may go beyond the threshold mileage of 30,000**
- **MLE with <u>censored</u> data fits a beta distributed CDF to sample data with tailed mileage**





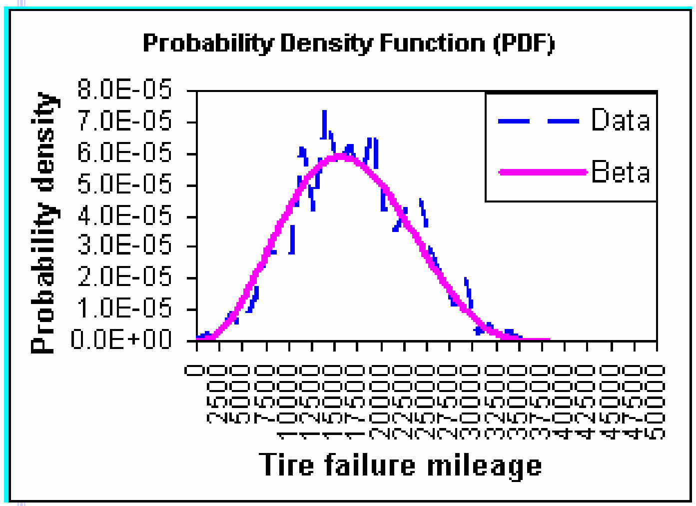


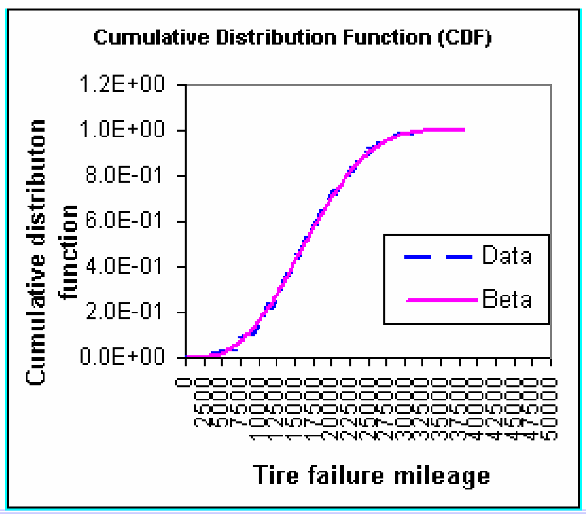


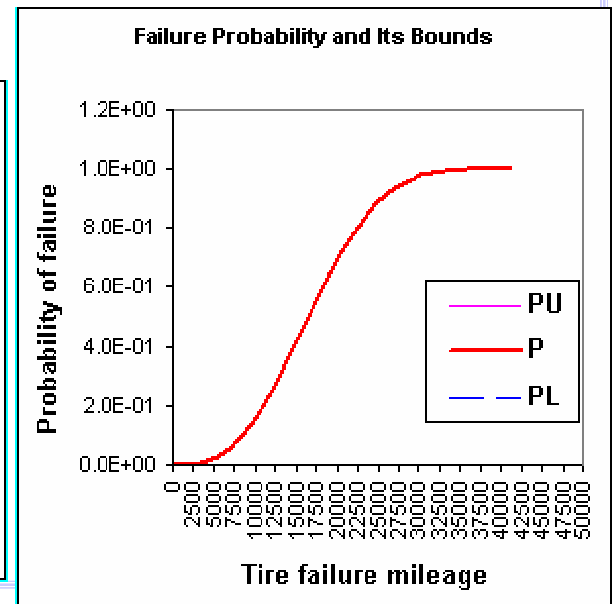
Censored MLE Approach (Method 1)

7. MLE with <u>uncensored</u> data considering tailed failure data

- **Includes both recorded failure data and "tailed" data**
- **Using MLE with uncensored data, a beta distributed CDF is fitted to the recorded and "tailed" data**
- **Failure probability is calculated**







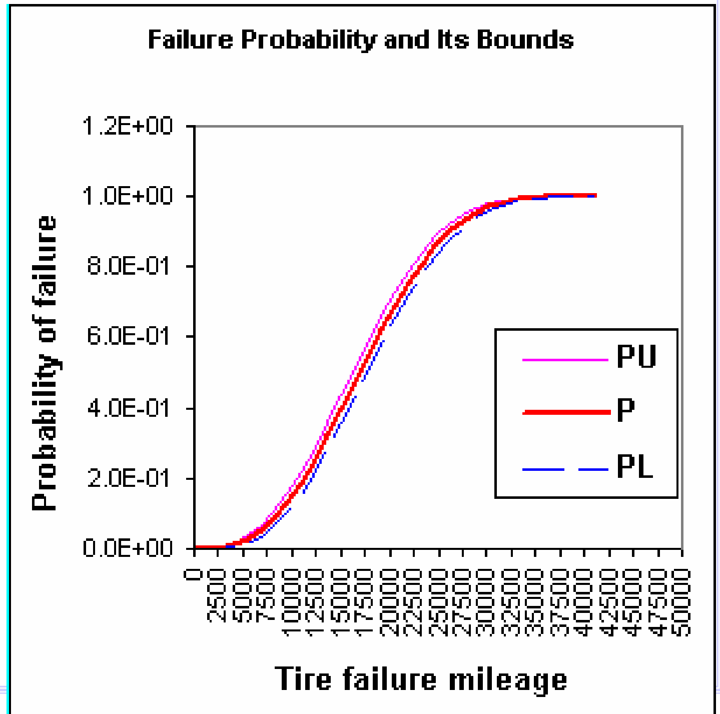
Censored MLE Approach (Method 1)

8. Failure probability bounds are calculated using Bootstrap

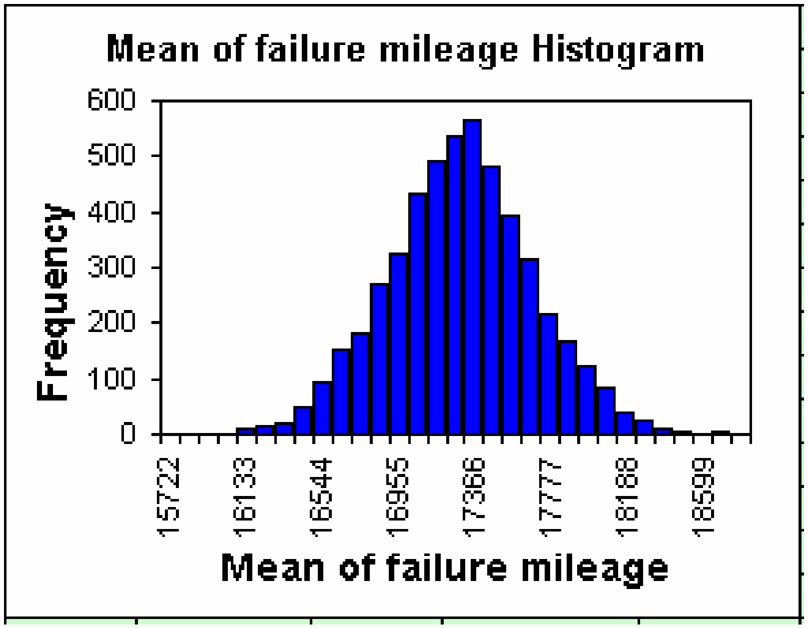
- **Both recorded and "tailed" data are used.**
- **5000 samples (sets of sample points) are randomly generated from the recorded and "tailed" sample.**
- **Failure probability bounds with confident level of 0.9 are calculated.**
- **Statistics of other parameters are provided (mean of failure mileage, std dev of failure mileage, parameters *p* and *q*, and probability of failure).**

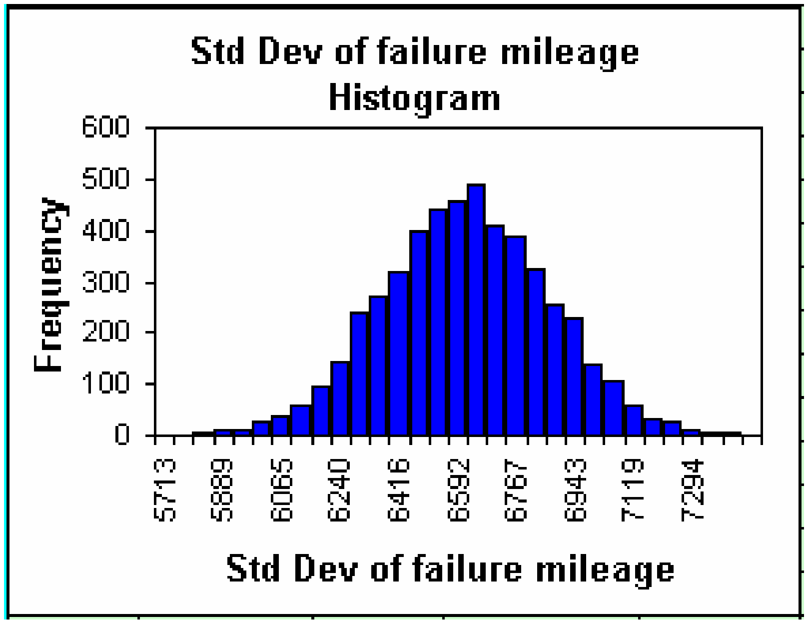
	J	K	L	M	N	O	P
1	Std Dev	Skewness	Kurtosis	Confidence Level	Resamplings		
2	6601	0.2184	-0.3738	0.9	5000		
3–4	ood)	Param Estimates (Explicit)		Bootstrap	Bayesian Update		
5	30	Distrib. Type	Beta	Mileage to failure<	*P*	*P*L	*P*U
6	Frequency	Data PDF	Data CDF	15000	3.90E-01	3.51E-01	4.29E-01
7	0	0.00E+00	0.0000	0	0.00E+00	0.00E+00	0.00E+00
8	1	2.59E-06	0.0036	1374	2.51E-04	8.17E-05	6.23E-04
9	0	0.00E+00	0.0036	2748	2.67E-03	1.24E-03	5.02E-03
10	4	1.04E-05	0.0178	4122	1.02E-02	5.72E-03	1.64E-02
11	3	7.77E-06	0.0285	5496	2.53E-02	1.62E-02	3.67E-02
12	4	1.04E-05	0.0427	6870	5.01E-02	3.53E-02	6.74E-02
13	10	2.59E-05	0.0783	8244	8.53E-02	6.45E-02	1.08E-01
14	13	3.37E-05	0.1246	9618	1.31E-01	1.05E-01	1.59E-01
15	16	4.14E-05	0.1815	10993	1.87E-01	1.55E-01	2.19E-01
16	18	4.66E-05	0.2456	12367	2.51E-01	2.16E-01	2.86E-01
17	22	5.70E-05	0.3238	13741	3.22E-01	2.84E-01	3.59E-01
18	19	4.92E-05	0.3915	15115	3.97E-01	3.58E-01	4.35E-01

Demo Data / Failure Data / **Data Analysis** / Beta Dist / Resampling Tab / Bayesia

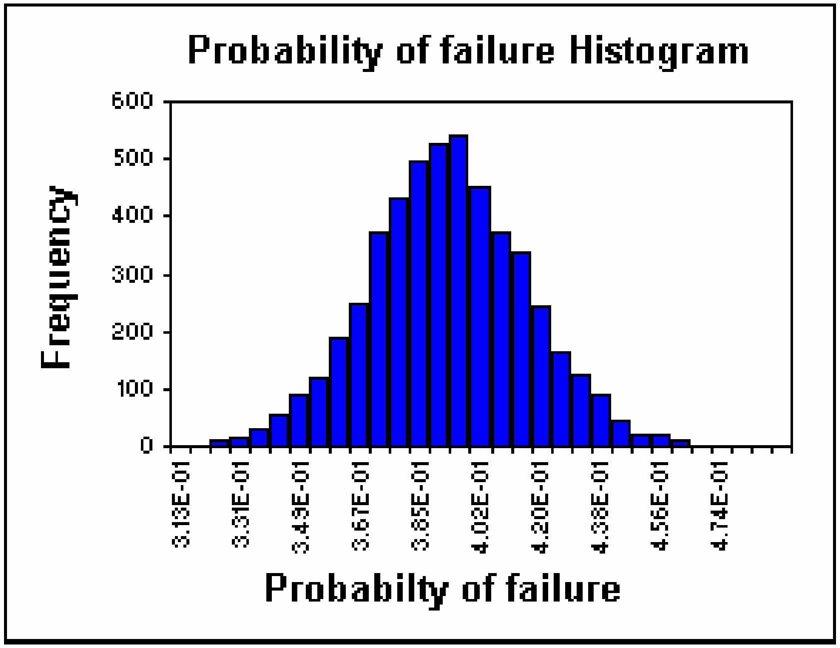


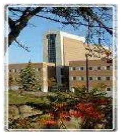
Censored MLE Approach (Method 1)





Censored MLE Approach (Method 1)

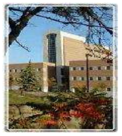




Bayesian Updating Approach (Method 2)

- **Specify “PRIOR” distribution**
- **Calculate “LIKELIHOOD” distribution**
- **Calculate “POSTERIOR” distribution**

Bayesian Updating Approach (Method 2)



1. Specify "PRIOR" distribution

- **"PRIOR source" <u>Option 0</u>: Uniform (non-informative) distribution**
- **"PRIOR source" <u>Option 3</u>: Normal distribution for each parameter -- Expert opinion**

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	DATA source	1	THEORY				PROCEDURE	Reset	PRIOR source	3			
2	Parameter	θ_1=A	θ_2=B	θ_3=p	θ_4=q		Step 1		Parameter	θ_1 = A	θ_2 = B	θ_3 = p	θ_4 = q
3	Active	0	0	1	1		Step 2		μ_θ	0	45000	3	10
4	θ_L	0	45000	1	1		Step 3	PRIOR	σ_θ	0	0	0.3	1.5
5	θ_U	0	45000	10	20		Step 4	LIKELIHOOD	Min_θ				
6	θ_{ref}	0	45000	3	5		Step 5	POSTERIOR	Max_θ				
7	N_{grid}	1	1	100	100				$\theta_{best\ estimated}$				
8	Precision	2	2	2	2								
9						You 're updating the parameter:		0	time:		NOTHING	is finished.	

Demo Data / Failure Data / Data Analysis / Beta Dist / Resampling Tab / **Bayesian Updating** / GA Solver / output generations / warnings

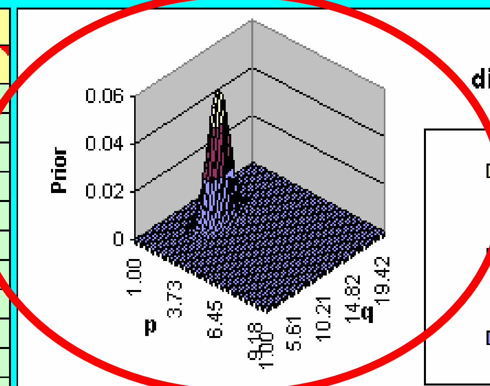
Bayesian Updating Approach (Method 2)

1. Specify "PRIOR" distribution (Cont'd)

- **"Updated Parameter Distribution Table" and 2-D Diagram**
- **"PRIOR source" option is automatically set to 1**

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**DATA source**	1	**THEORY**				**PROCEDURE**	Reset	**PRIOR source**	1			
2	**Parameter**	θ_1=*A*	θ_2=*B*	θ_3=*p*	θ_4=*q*		**Step 1**		**Parameter**	θ_1 = *A*	θ_2 = *B*	θ_3 = *p*	θ_4 = *q*
3	Active	0	0	1	1		**Step 2**		μ_θ	0	45000	3	10
4	θ_L	0	45000	1	1		**Step 3**	PRIOR	σ_θ	0	0	0.3	1.5
5	θ_U	0	45000	10	20		**Step 4**	LIKELIHOOD	Min_θ				
6	θ_{ref}	0	45000	3	5		**Step 5**	POSTERIOR	Max_θ				
7	N_{grid}	**1**	**1**	**100**	**100**				$\theta_{best\ estimated}$				
8	Precision	2	2	2	2								
9						*You're updating the parameters*		1	*st time:*	**PRIOR**		*is finished.*	
10													

	Updated Parameter Distribution Table								
12	Grid Point ID	θ_1=*A*	θ_2=*B*	θ_3=*p*	θ_4=*q*	**Prior**	**Likelihood**	**Posterior**	**"CDF"**
13	1	0	45000	1	1	1.9149E-19			
14	2	0	45000	1	1.191919	4.0924E-19			
15	3	0	45000	1	1.383838	8.6043E-19			
16	4	0	45000	1	1.575758	1.7797E-18			
17	5	0	45000	1	1.767677	3.6212E-18			
18	6	0	45000	1	1.959596	7.2487E-18			
19	7	0	45000	1	2.151515	1.4274E-17			
20	8	0	45000	1	2.343434	2.7653E-17			
21	9	0	45000	1	2.535354	5.2701E-17			
22	10	0	45000	1	2.727273	9.[illegible]606E-17			
23	11	0	45000	1	2.919192	1.8224E-16			



Demo Data / Failure Data / Data Analysis / Beta Dist / Resampling Tab / **Bayesian Updating** / GA Solver / output generations / warnings

Bayesian Updating Approach (Method 2)

2. Calculate “LIKELIHOOD” distribution

- **“Updated Parameter Distribution Table” and 2-D Diagram**

	B	C	D	E
5	0	45000	10	20
6	0	45000	3	5
7	1	1	100	100
8	2	2	2	2

Step 4 LIKELIHOOD

Step 5 POSTERIOR

Min_θ

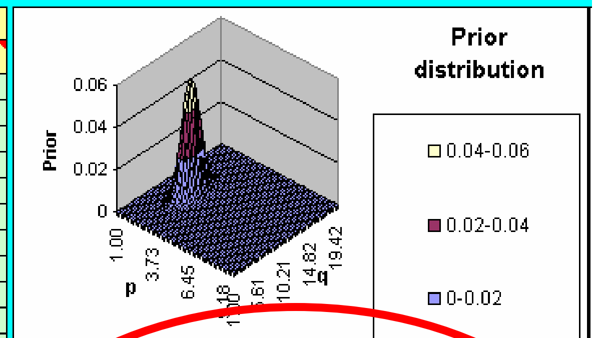
Max_θ

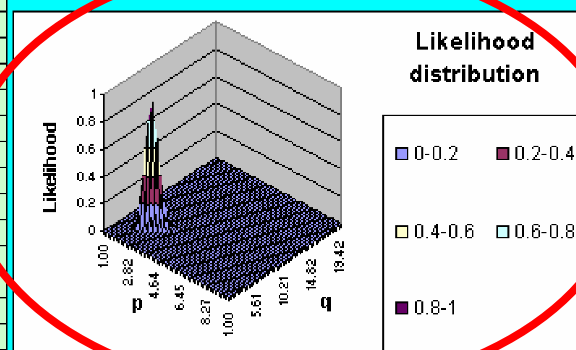
$\theta_{best\ estimated}$

You're updating the parameters 1 *st time:* LIKELIHOOD *is finished.*

Updated Parameter Distribution Table

	$\theta_1=A$	$\theta_2=B$	$\theta_3=p$	$\theta_4=q$	Prior	Likelihood	Posterior	"CDF"
13	0	45000	1	1	1.9149E-19	0		
14	0	45000	1	1.191919	4.0924E-19	0		
15	0	45000	1	1.383838	8.6043E-19	0		
16	0	45000	1	1.575758	1.7797E-18	0		
17	0	45000	1	1.767677	3.6212E-18	0		
18	0	45000	1	1.959596	7.2487E-18	0		
19	0	45000	1	2.151515	1.4274E-17	0		
20	0	45000	1	2.343434	2.7653E-17	0		
21	0	45000	1	2.535354	5.2701E-17	0		
22	0	45000	1	2.727273	9.8806E-17	0		
23	0	45000	1	2.919192	1.8224E-16	0		
24	0	45000	1	3.111111	3.3067E-16	0		
25	0	45000	1	3.30303	5.9024E-16	0		
26	0	45000	1	3.494949	1.0365E-15	0		
27	0	45000	1	3.686869	1.7905E-15	0		
28	0	45000	1	3.878788	3.0429E-15	0		
29	0	45000	1	4.070707	5.0873E-15	0		
30	0	45000	1	4.262626	8.3672E-15	0		
31	0	45000	1	4.454545	1.3538E-14	0		
32	0	45000	1	4.646465	2.1549E-14	0		
33	0	45000	1	4.838384	3.3744E-14	0		
34	0	45000	1	5.030303	5.1982E-14	0		
35	0	45000	1	5.222222	7.8776E-14	0		
36	0	45000	1	5.414141	1.1744E-13	0		
37	0	45000	1	5.606061	1.7225E-13	0		





Demo Data / Failure Data / Data Analysis / Beta Dist / Resampling Tab / **Bayesian Updating** / GA Solver / output generations / warnings

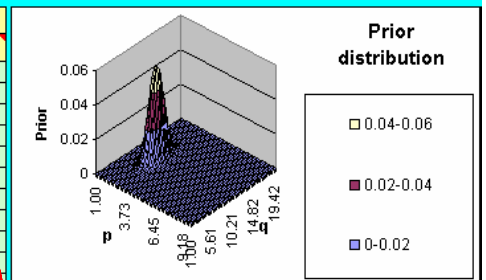
Bayesian Updating Approach (Method 2)

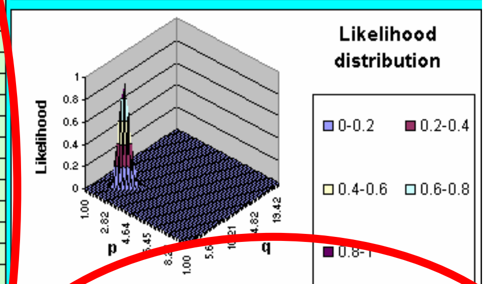
3. Calculate "POSTERIOR" distribution

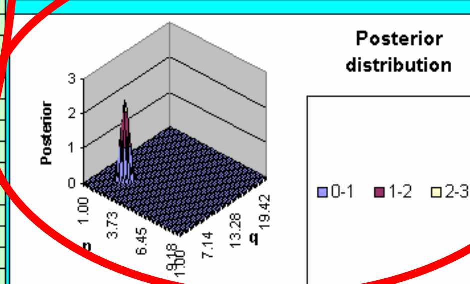
- **"Updated Parameter Distribution Table" and 2-D Diagram**

	B	C	D	E	F	G	H	I	J	K	L	M	N
6	0	45000	3	5		Step 5	POSTERIOR	Max_θ	0	45000	3.818182	7.1414142	
7	1	1	100	100				$\theta_{best\ estimated}$	0	45000	3.090909	5.6060606	
8	2	2	2	2									
9					*You're updating the parameters*		1	*st time:*	POSTERIOR		*is finished.*		
10													

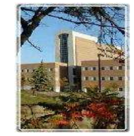
	Updated Parameter Distribution Table							
12	θ_1=*A*	θ_2=*B*	θ_3=*p*	θ_4=*q*	Prior	Likelihood	Posterior	"CDF"
13	0	45000	1	1	1.9149E-19	0	0	0
14	0	45000	1	1.191919	4.0924E-19	0	0	0
15	0	45000	1	1.383838	8.6043E-19	0	0	0
16	0	45000	1	1.575758	1.7797E-18	0	0	0
17	0	45000	1	1.767677	3.6212E-18	0	0	0
18	0	45000	1	1.959596	7.2487E-18	0	0	0
19	0	45000	1	2.151515	1.4274E-17	0	0	0
20	0	45000	1	2.343434	2.7653E-17	0	0	0
21	0	45000	1	2.535354	5.2701E-17	0	0	0
22	0	45000	1	2.727273	9.8806E-17	0	0	0
23	0	45000	1	2.919192	1.8224E-16	0	0	0
24	0	45000	1	3.111111	3.3067E-16	0	0	0
25	0	45000	1	3.30303	5.9024E-16	0	0	0
26	0	45000	1	3.494949	1.0365E-15	0	0	0
27	0	45000	1	3.686869	1.7905E-15	0	0	0
28	0	45000	1	3.878788	3.0429E-15	0	0	0
29	0	45000	1	4.070707	5.0873E-15	0	0	0
30	0	45000	1	4.262626	8.3672E-15	0	0	0
31	0	45000	1	4.454545	1.3538E-14	0	0	0
32	0	45000	1	4.646465	2.1549E-14	0	0	0
33	0	45000	1	4.838384	3.3744E-14	0	0	0
34	0	45000	1	5.030303	5.1982E-14	0	0	0
35	0	45000	1	5.222222	7.8776E-14	0	0	0
36	0	45000	1	5.414141	1.1744E-13	0	0	0
37	0	45000	1	5.606061	1.7225E-13	0	0	0
38	0	45000	1	5.79798	2.4852E-13	0	0	0
39	0	45000	1	5.989899	3.5275E-13	0	0	0
40	0	45000	1	6.181818	4.9257E-13	0	0	0
41	0	45000	1	6.373737	6.7664E-13	0	0	0
42	0	45000	1	6.565657	9.1439E-13	0	0	0
43	0	45000	1	6.757576	1.2156E-12	0	0	0
44	0	45000	1	6.949495	1.5899E-12	0	0	0
45	0	45000	1	7.141414	2.0455E-12	0	0	0
46	0	45000	1	7.333333	2.5891E-12	0	0	0
47	0	45000	1	7.525253	3.2239E-12	0	0	0
48	0	45000	1	7.717172	3.9491E-12	0	0	0
49	0	45000	1	7.909091	4.7589E-12	0	0	0
50	0	45000	1	8.10101	5.6417E-12	0	0	0







Bayesian Updating Approach (Method 2)



3. Calculate "POSTERIOR" distribution (Cont'd)

- **Updated "PRIOR source"**
  - **Best estimated (most probable) parameters (peak point of posterior distribution)**
  - **Means and stand. deviations of parameters; Obtained by sampling posterior 5000 times**
  - **Ranges (min, max) of parameters**

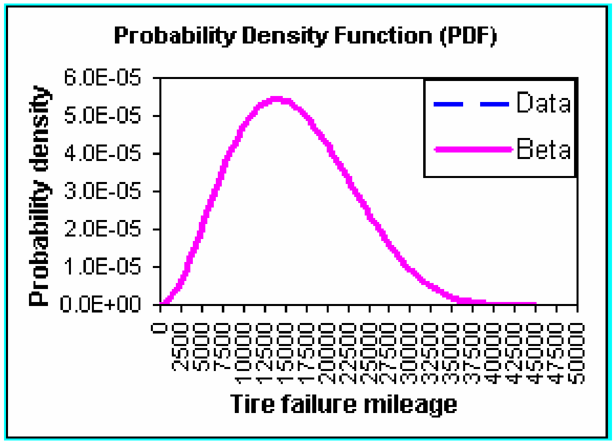
	B	C	D	E	F	G	H	I	J	K	L	M
1	1	THEORY				PROCEDURE	Reset	PRIOR source	1			
2	θ_1=A	θ_2=B	θ_3=p	θ_4=q	←	Step 1		Parameter	θ_1 = A	θ_2 = B	θ_3 = p	θ_4 = q
3	0	0	1	1		Step 2	→	μ_θ	0	45000	3.080527	5.3955252
4	0	45000	1	1		Step 3	PRIOR	σ_θ	0	0	0.200531	0.4467504
5	0	45000	10	20		Step 4	LIKELIHOOD	Min_θ	0	45000	2.363636	3.6868687
6	0	45000	3	5		Step 5	POSTERIOR	Max_θ	0	45000	3.818182	7.1414142
7	1	1	100	100				$\theta_{best\ estimated}$	0	45000	3.090909	5.6060606
8	2	2	2	2								
9					*You're updating the parameters*		1	*st time:*	POSTERIOR		*is finished.*	

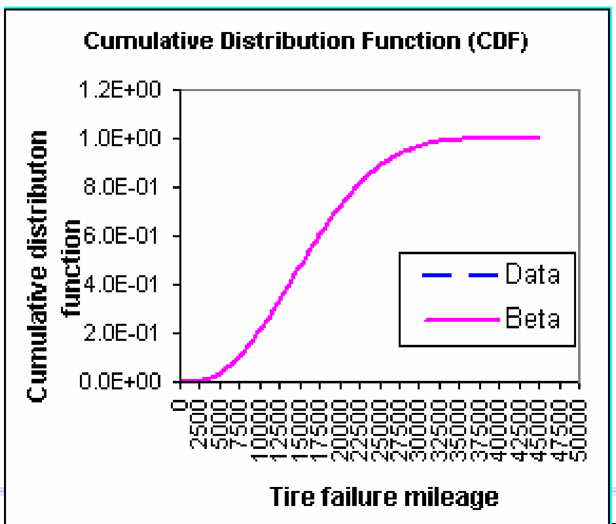
Demo Data / Failure Data / Data Analysis / Beta Dist / Resampling Tab / **Bayesian Updating** / GA Solver / output generations / warnings

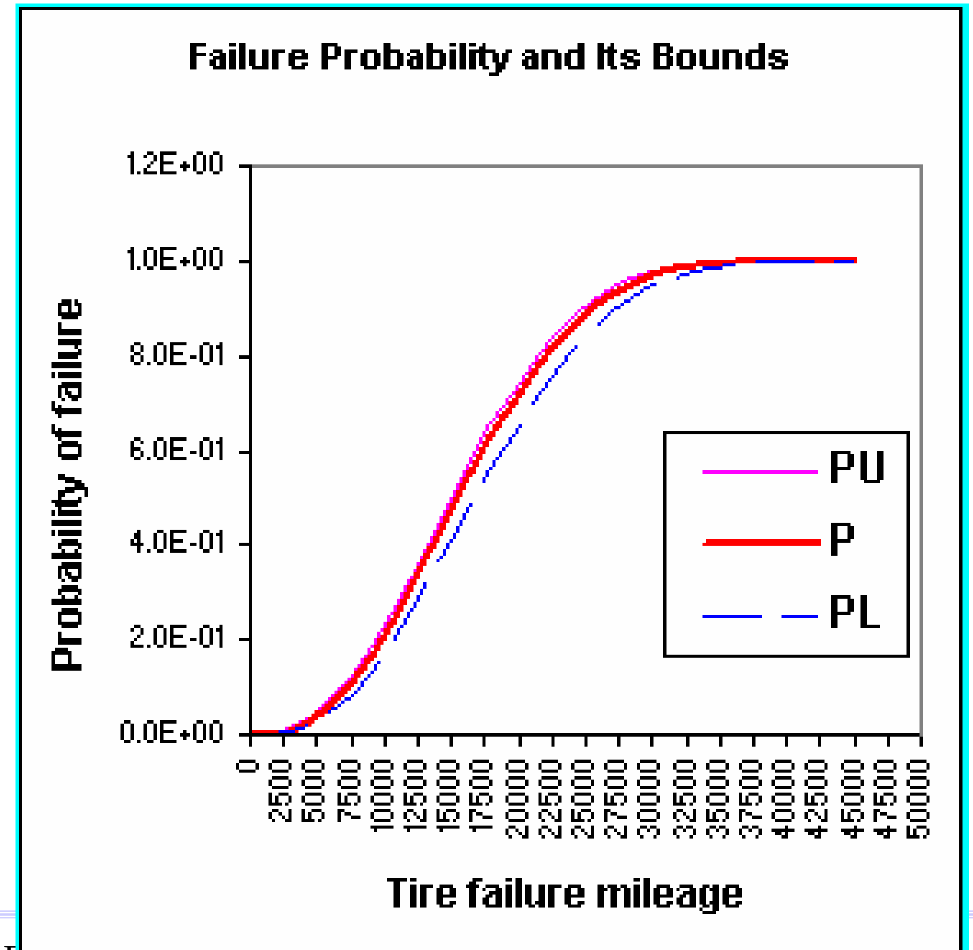
Bayesian Updating Approach (Method 2)

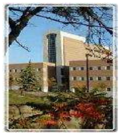
3. Calculate "POSTERIOR" distribution (Cont'd)

- **PDF and CDF of Failure Probability and Its Bounds (sampling posterior 5000 times)**





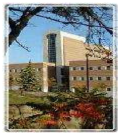




Summary

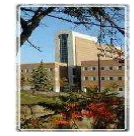
Two methods have been presented to estimate statistics of Time Between Failures (TBF) using limited, censored data

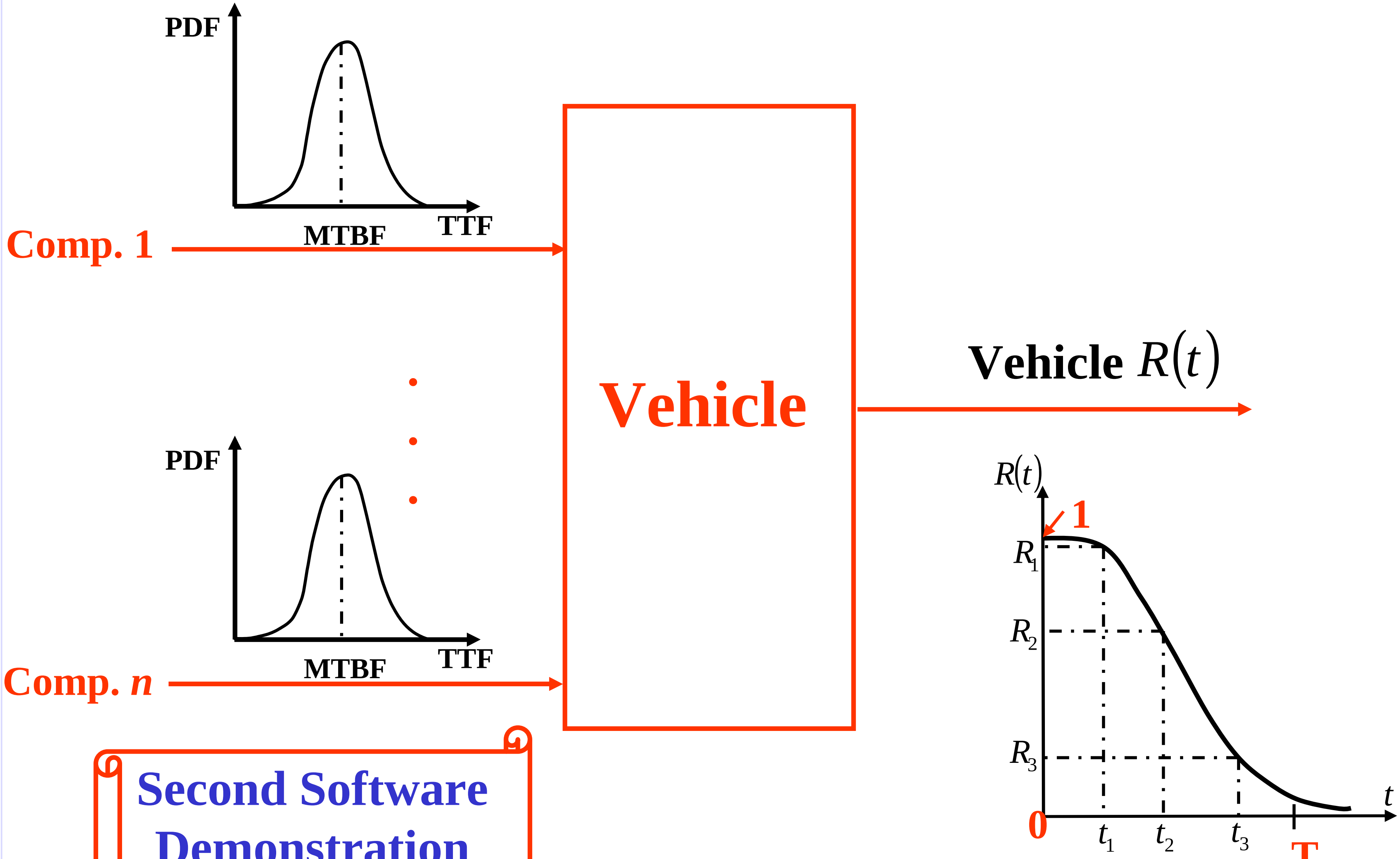
- **Censored MLE approach (Method 1)**
- **Bayesian updating approach (Method 2)**
    - ✓ **“Enhances” data with expert opinion**



Potential Developments in Durability, Reliability, Availability and Maintainability

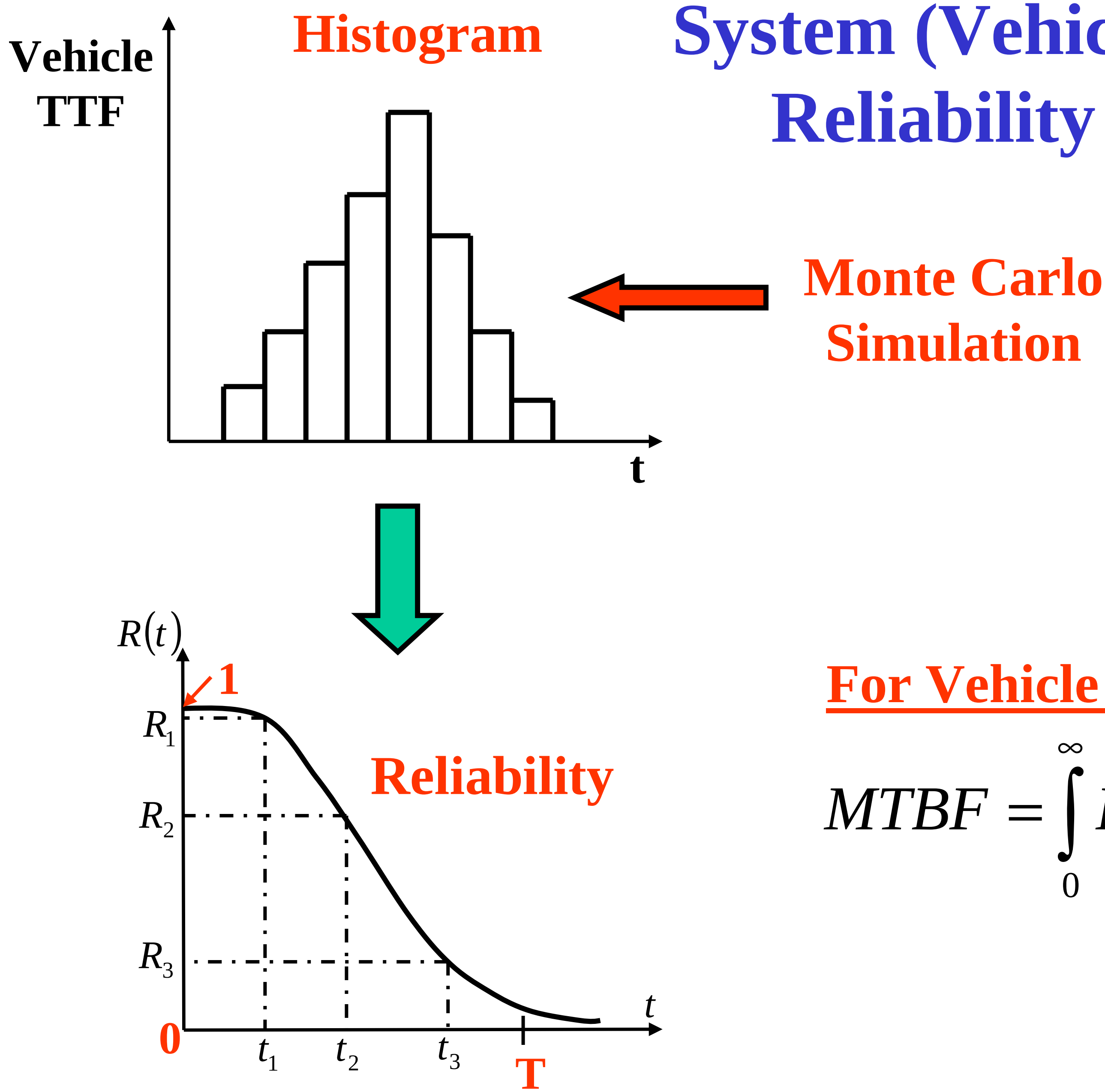
System (Vehicle) Reliability





Second Software Demonstration

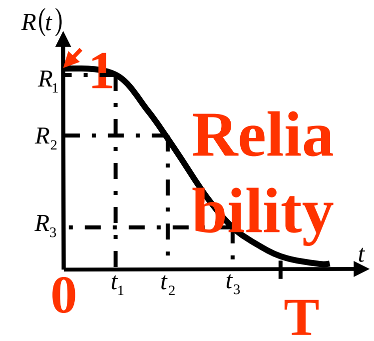
System (Vehicle) Reliability



For Vehicle :

$$MTBF = \int_{0}^{\infty} R(t)\,dt$$

Reliability Allocation



Specify system (vehicle) reliability

Optimization

Determine required reliability of EACH component

This optimization problem DOES NOT have a unique solution

Reliability Allocation

One way to get a unique solution is to trade-off reliability and associated cost

$$\min_{\underline{R}_{comp}} \quad Cost$$

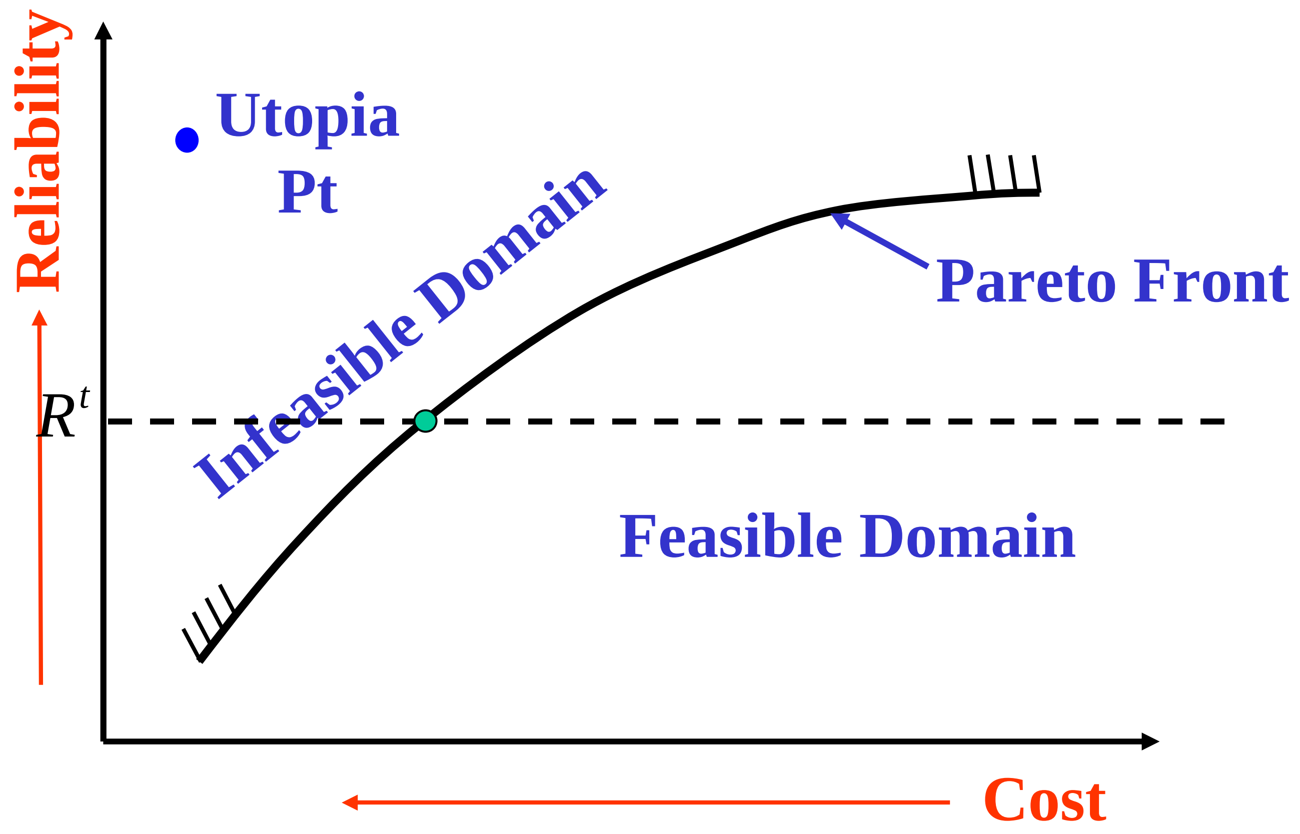
Target system reliability

s. t. $\quad System \quad \mathrm{Re}\,liability = R^t$

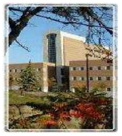
By varying R^t , we get the so called "Pareto Frontier."

Reliability vs Risk of Failure (Cost)

We want to maximize Reliability and simultaneously minimize Risk of failure (cost)



Putting it All Together !!!



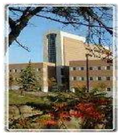
p_{MFFOP} : **Probability of achieving MFFOP**

Determine component hazard rates to:

- **Max Reliability**
- **Min Cost**
- **Max Availability**
- **Max MFFOP**
- **…**

Multi-Objective Optimization

$$Availability = \frac{E[Uptime]}{E[Uptime] + E[Downtime]}$$



Q & A